Notes on the Discount rate

The choice of discount rate for project economic analysis has been the subject of considerable debate in both theoretical and operational terms. In theory the discount rate for the analysis of any individual project will depend on how project financing affects the rest of the economy. In principle funds used by a project may be diverted from consumption or from investment in other projects or a combination of the two. Therefore the choice of discount rate will vary depending upon what is assumed about project financing.

Social Time Preference

Where funds are diverted from consumption the appropriate discount rate is a social time preference rate (STPR), which captures society's preference for receiving consumption in the present rather than at different times in the future, with the discount rate capturing the decline in value of distant consumption. Different specifications of this rate are possible but the most common derivation is to assume that

- future consumption is less valuable because as incomes grow over time it will be received when individuals are better-off than they are in the present (a marginal utility effect)
- future consumption is less valuable because of catastrophic unpredictable risk either to individuals or society (a pure time preference effect).
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The standard formula is

$$STPR = g.n + p \qquad (1)$$

Where g is the growth of per capita consumption, n is the elastcity of marginal utility of consumption and p is measure of pure time preference

Attempts to quantify this version usually come to relatively low rates of between 1% and 3%, with the main factors driving these estimates what is assumed about future growth of consumption per capita and the extent to which this influences future valuation. This latter variable termed technically the elasticity of the marginal utility of consumption is often set at 1.0 so, with the latter pure time preference often set a very low rate, that what is assumed about consumption growth becomes the main term in the derivation.

The UK Treasury rate as set out in the Green Book is for a STPR, based on g at 2%, n at 1.0 and p at 1.5%, give an overall discount rate of 3.5%

It is important to stress that in project analysis use of a social time preference rate (1% to 3%) in appraisals as the test rate which the EIRR must pass will only be correct theoretically in situations where there is no savings shortage; in other words, where the system of financial intermediation can supply adequate funds for all projects with a return above this test rate. In most borrower developing countries it is safe to assume this will not be the case. Where there is a savings shortage by definition savings will be relatively scarce and thus more valuable than consumption. This means that an economic analysis which uses the STPR as the test rate must ensure that all savings used by a project when it is financed and all savings created by a project when part of its annual surplus is reinvested must be revalued by a savings premium which reflects this scarcity. This savings premium is difficult to calculate because it requires assumptions about not just the STPR, but also the marginal returns on new investment (that is the opportunity cost of capital), the reinvestment rate out of the annual surplus of new investment, and the life of new investment. Based on simplifying assumptions formulae exist to approximate the savings premium but it is highly sensitive to each of these parameters.

Assuming an infinite life of investment and constant values for all parameters the value of savings (P) equals

$$(1-s).q/(i-s.q), \qquad (2)$$

where q is the OCC, i is STPR and s is savings and reinvestment out of the annual return on investment.

Thus where for example q = 0.12, i = 0.03 and s = 0.10, P will be 6. The premium increases the greater the gap between the OCC (q) and STPR (i).If s rises to 20%, value of savings increases to 16.

Because in part because of this uncertainly in practice it is relatively rare to see a saving premium applied in appraisals. However the important theoretical point is that a low test discount rate based on an STPR of 1%-3% is not necessarily a low hurdle for a project to pass, where it is applied in conjunction with a saving premium that incorporates an opportunity cost of capital (OCC) in the range of 10%-12%. Unless the savings effect of a project is small the test in fact will be similar to that posed by a relatively high opportunity cost discount rate of 10% to 12%.

Opportunity cost

Where funds for a project displace investment and where it is assumed all of the annual surplus generated by a project is available for re-investment the theoretically correct discount rate is the return on the investment that the project displaces, which is the opportunity cost of capital (OCC). In situations of capital scarcity by definition this will exceed the STPR often by a relatively large number of percentage points. Various sources of information can be used to obtain an estimate of the rate of return on investments in an economy. At one level, national income accounting data converted to economic prices can be used to calculate a national level profit estimate. This should then be compared with a national capital stock estimate. At a second level, sector or corporate data is used to estimate a weighted average return in financial prices. This return then needs to be converted to economic prices using national parameter estimates. At a third level, studies of recently accepted and rejected projects are used to identify a minimum economic rate of return that appears acceptable to the government. These approaches generally give a range of estimates for the economic OCC, however in general OCC estimates tend to be in the 8% to 15% range.

These figures may appear high as real rates of return when compared with financial yields, however they are not inconsistent with what is known about incremental capital-output ratios (ICOR) for fast growing economies. An ICOR is a ratio of investment to change in aggregate value added in an economy. An ICOR of 3.0 for example (which is not particularly low) implies a ratio of incremental value-added to investment of 0.33. With a labour content in value-added of 30%, this implies an annual return on investment of over 20%.

Weighted average approach

Where a project affects both consumption and the funds available for investment the theoretically correct approach is to use a weighted average of the STPR and OCC rates as the discount rate. Again in theory where a project is large enough so that is demand for funds affects the prevailing market interest rate, the weights will be determined by the responsiveness of savings (creating a change in consumption) and investment (creating a change in other projects) respectively, to a small change in the interest rate. This responsiveness is the elasticity of savings and investment, respectively to the interest rate. Empirical analysis generally suggests that investment is much more responsive to a change in interest rates than is aggregate savings (although forms of savings may change more than the total), so that the expectation is that in a weighted average comparison the weighted discount rate will be closer to the OCC than the lower STPR. In practice however whilst weighted costs of capital are a standard approach to discounting in financial analysis, it is relatively uncommon to see weighted discount rates in economic appraisals, partly because of the uncertainty about how the actual distribution of project financing impacts on savings/consumption and investment and partly because as we discuss below it is more usual to apply the discount rate as a rationing device to allocate a fixed budget.

Rationing discount rate

The approach adopted by multilateral development banks (MDBs) is to use a single test discount rate for consistency across projects and countries. The logic is that the bank has a fixed amount of funds for project lending each year and that this corresponds to a limited investment budget. 12% is taken as the return on a marginal project, so that if the project under consideration does not pass this test there will be another project that will. 12% thus represents the OCC of bank funds. This relatively high rate is based on the assumption that real returns to capital are high in borrower countries where there is a scarcity of funds for investment. Most borrower governments in Asia and MDBs use discount rates at a similar level. This is fully consistent with the fact that it also reports that most governments in higher income economies, where financial intermediation is far more developed and by implication there is no savings shortage, use much lower rates corresponding to STPRs.

Use of single rate such as 12% is based on the assumption that there is single fixed investment budget that is fungible between sectors and countries. Thus if a project in sector X country j does not pass the 12% test, the assumption is that the funds not committed to the project go either to a project in the same sector in country k, or to another a project in sector Y in the original country j. Thus efficiency across the whole portfolio requires use of a single test rate to ensure the fixed funds are allocated efficiently.

However if there is a fixed budget either per sector or per country this approach will be inappropriate, since the opportunity cost of funds committed to a project will be given by the marginal project in the same sector (where it is the sector budget that is fixed) or the marginal project in the same country (where it is the country budget that is fixed). In theory, where there is immobility of capital within a country, so that funds do not flow freely between sectors, then there is no reason why marginal financial returns should be equalized across sectors. Similarly, where as in practice, externalities and taxes/subsidies differ in their incidence between sectors marginal economic returns will also differ between sectors, even in the presence of capital mobility.

International immobility of capital combined with different levels of capital scarcity mean that we should also expect marginal returns on investment to differ between countries. In principle therefore with fixed country budgets for borrowers, strictly appraisals should be conducted with country-specific discount rates, reflecting national OCCs. If this approach is adopted however a national rationing discount rate should be estimated as part of country macro-economic assessments and used consistently for all projects in that country. It should not be estimated ad hoc as part of the appraisal of an individual project.

Declining discount rate

An important argument for a declining (hyperbolic) discount rate from the academic literature based on increasing uncertainty over time has been adopted by the UK Treasury in the UK Green Book for the assessment of public sector projects. This uses a STPR which declines after 30 years. The rationale is that the discount rate implies a discount factor (DF) for year t given as

$DF = 1/(1 + i)^t$ where i is the STPR and t is the year.

By rearranging $i = (1/DF^{1/t} – 1)$

In theory in the presence of uncertainty a probability weighted DF must be used to a give a ‘certainty equivalent’ DF (CEDF), so

$CEDF = p1.DF_1 + p2.DF_2.......pn.DF_{n...}$ for all n possible values of i, and where p is the probability of occurrence (so p1 + p2 … pn = 1.0)

If CEDF replaces DF in the formula for i then $i = (1/CEDF^{1/t} – 1)$, which allows the STPR to be recalculated with probabilities assigned to different rates. It can be shown that as t rises (that is as time passes) calculating i in this way leads to a fall in its value, thus justifying a declining STPR. A numerical illustration is given below.

Discount rate	probability	Discount Factor				
		year	10	50	100	200
			0.82034	0.37152	0.13803	0.01905
0.02	0.4		8	8	3	3
			0.67556	0.14071		0.00039
0.04	0.3		4	3	0.0198	2
			0.55839	0.05428	0.00294	
0.06	0.3		5	8	7	8.69E-06
			0.69832	0.20711	0.06203	0.00774
CEDF			7	1	7	1
CE			0.03655	0.03199		0.02460
STPR			9	1	0.02819	4

Three possible discount rates are identified 2%, 4% and 6% with respective probabilities assumed to be 40%, 30% and 30% respectively. The probability weighted or certainty equivalent discount factor (CEDF) is shown for the years 10, 50, 100 and 200 and the certainty equivalent discount rates (CE STPR) change each year and fall gradually over time, for example from 3.6% in year 10 to 3.2% in year 50, to 2.8% in year 100. As the time horizon lengthens the rate falls towards the lowest of the possible rates and in this case converges on 2%.

This approach provides a justification for variable and declining discount rates and is one way of addressing a bias in the treatment of long-term effects, such as positive or negative environmental impacts. However it is difficult to implement in any agreed way. A relatively simple approach for long-lasting projects, which is put forward in the UK Treasury Green Book is to use a standard STPR of 3.5% up to year 30 and then apply declining rates, based on the logic in the example above. However instead of changes in rates each year a single rate is used for blocks of years as set out below

Years 31-75

3.0%
Years 76 -125 2.5%
Years 126 -200 2.0%
Years 201 -300 1.5%
Years >301 1.0%.
This approach is difficult to reconcile with a single opportunity cost rationing rate (where probability does not enter into the calculation) unless there is clear evidence of the declining productivity of investment over time. Further as explained above and applied in the Treasury Green Book it will have only a relatively modest differential effect on the relative valuation of long-term effects, since the rate of decline in the discount rate is itself only modest.

Health and discounting

Use of a rationing discount rate such as 12% implies that all project effects can be converted into monetary equivalents, which is clearly a strong assumption of some types of project. This has emerged as a contentious issue for sectors such as health and the environment, because of the difficulty of putting monetary values on mortality and morbidity effects. The alternative is to apply cost effectiveness analysis. This requires quantifying the health impact of a project either in terms simple measures like bed nights or patients treated or in more sophisticated ways like disability adjusted years of life saved. These annual health impacts must be compared with the cost of the project spread over its life and thus requires a comparison over time. As there is a delay in receiving health benefits discounting becomes relevant. However because health benefits are not valued directly in monetary terms and are not treated as income that can potentially be reinvested an opportunity cost discount rate is not appropriate. The cost of delay can be interpreted as the time preference of individuals to receive health benefits sooner rather than later, so that cost effectiveness calculations in Health normally are conducted at the STPR.

The environment and discounting

The environment is the other hard to value area where the application of a standard opportunity cost rate may be misleading. There is a theoretical argument that if a STPR is to be applied then a separate discount rate is justified for environmental effects. This follows from the formula in equation (1) since if environmental resources are becoming more scarce then unlike consumption in the aggregate (g in the formula) in per capita terms use of environmental resources is falling not rising. Declining marginal utility in response to rising consumption means that consumption in the future, when consumers are better off, is worth less than the same amount of consumption today (as reflected by the product of g and n in the formula). However if the environment is becoming scarcer in the future, there will be no decline in the marginal utility associated with the use of environmental assets. Following this logic the first term in equation (1) drops out leaving only the pure time preference effect based on risk of death or catastrophe (p). In practice special environmental discount rates rationalised in this way are normally in the 0.5% to 1.5% range, although the approach remains controversial.

Use of dual discount rates with one for non-environmental and a separate lower rate for environmental effects is difficult to reconcile with the application of an opportunity cost discount rate. The alternative way of addressing any growing scarcity of environmental resources is through an adjustment to the relative valuation of environmental costs and benefits in constant prices. Growing scarcity should be reflected in an annual rise relative to general inflation. Arithmetically where all other prices are kept constant a rise in the price of any specific item is equivalent to discounting the future value of that item at a lower discount rate than the standard opportunity cost rate.

Where the assumed annual real rate of price increase (that is the projected increase relative to inflation) is θ and the opportunity cost discount rate is q the implied adjusted discount rate is approximately $q - \theta$. For example, where 12% is the opportunity cost rate and the projected real increase in the value of an environmental effect is 6% annually then if the environmental effect is increased by 6% each year but then discounted by 12% this is equivalent to discounting by 5.6%, since in the first year the environmental value is multiplied by 1.06 and then divided by 1.12 to give a discount factor of 0.9464. A discount factor of 0.9464 implies a discount rate of 5.6% as 0.9464 = 1.0566. In principle it would be possible to adjust different environmental effects by different rates of real increase in value which would imply a series of differential environmental discount rates. In practice it will be difficult to make such real price projections with any degree of certainty and only approximate adjustments as part of sensitivity analysis are recommended, unless firm information is available.